AIAA Journal
Vol. 51, No. 2, February 2013

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Aerodynamic Estimation of Annular Wings Based on Leading-Edge Suction Analogy

Adnan Maqsood*
National University of Sciences and Technology, Islamabad 44000, Pakistan
and
Tiauw Hiong Go†
Nanyang Technological University, Singapore 639798, Republic of Singapore

DOI: 10.2514/1.J051914

Nomenclature

AR	=	aspect ratio
C_L	=	coefficient of lift
$C_{L\alpha}$	=	wing lift-curve slope
$C_{l\alpha}$	=	section lift-curve slope
K_p	=	coefficient of lift
K_v	=	coefficient of lift
$(L/D)_{\max}$	=	maximum lift-to-drag ratio
α	=	angle of attack

I. Introduction

ANNULAR wings belong to the class of nonplanar wing configurations and have gained significant popularity recently [1–5] among the designers of unmanned air vehicles (UAVs) and micro air vehicles (MAVs). The lift increment of the annular wing from any planar configuration of same aspect ratio is well known. Moreover, for small aspect ratios, a significant improvement in $(L/D)_{\max}$ is observed. Not surprisingly, the increased popularity of annular wings has been driven by their application in ducted-fan UAVs. Annular shroud around the fan helps to increase the static and dynamic thrusts produced, and when the shroud is designed as an annular wing, it also acts as a lifting surface in the forward flight. Typical examples of UAVs with annular wings are the Honeywell MAV by AVID, LLC [1,2], the Singapore Technologies (ST) fantail system [3], and the GoldenEye UAV [4]. The application of the annular wing on passenger aircraft has also been conceptualized. Although the work related to annular wings can be traced back to [6] in 1947, the generic theoretical aerodynamic models of such wings have not been fully studied and have not received significant attention from technology protagonists beside its recognized advantages.

Ribner [6] analytically derived the lift characteristics of annular wings and concluded that their lift-curve slope is twice that of the flat-plate elliptical wings of the same aspect ratio. The seminal work in [7] represented the first formal experimental investigations on annular wings by a varying aspect ratio and comparing their lift-curve slope with various theoretical models. The investigation included the longitudinal aerodynamic characteristics of five annular wings of aspect ratio $(AR) = 1/3$, $2/3$, 1.0, 1.5, and 3.0 with equal projected areas and the Clark-Y airfoil cross section (thickness-to-chord ratio of 11.7%). The Reynolds numbers of the experiment varied, $Re =$ 0.704–2.11 million, due to the variation in the root chords of the wings. The results indicated that the lift-curve slopes of the annular wings were about twice that of the lift-curve slopes for planar rectangular wings having the same aspect ratio, which confirmed the findings of [6]. Recently, Traub [5] has revisited the topic by carrying out the experimental investigation on the baseline annular wing aerodynamics and studied the effects of gap and aspect ratio. The findings for aspect ratio variation coincide with the results in [7], but no analytical approximation of the lift behavior is attempted. Demasi [8] has developed a theoretical model of the minimum-induced drag prediction for annular wings based on the lifting line theory and the small perturbation acceleration method.

A simplified concept of leading-edge suction analogy was proposed in [9] for the low-aspect ratio planforms (specifically delta wings) in the late 1960s. The aerodynamic surfaces at moderate angles of attack suffer from flow separation at or near leading edge, which significantly alters the pressure distribution on the upper surface. The approach assumes that the total lift in the prestall regime can be calculated as the sum of the component of lift from potential flow (based on fully attached pressure distribution) and the other component associated with the separated leading-edge vortices. The total force on the wings (prior to stall) associated with the pressure required to stabilize the separated vortices is equivalent to the leading-edge suction force to keep the flow around the wings attached. This theory was extended in [10] to other planform shapes, such as rectangular wings in subsonic and supersonic regimes. Suction analogy for the side-edge vortices was also proposed in this work. Recently, this approach has been used to study the aerodynamic characteristics of different low-aspect ratio planform shapes at the low Reynolds numbers in [11,12]. Specifically, the aerodynamic effect of pressure-induced forces and vortex-induced forces are parameterized and can be observed explicitly for various planform shapes and aspect ratios.

The vortex generated from leading-edge flow separation is also observed in the annular wings. In [5], during the surface flow visualization, a significant region of separation is observed on top of the annular wing with an increasing angle of attack. The lower surface also showed evidence of large scale separation in the vicinity of the stall. Such separation leads to vortex generation and, consequently, a suction force that will contribute to lift as in the planar wing. Therefore, the influence of leading-edge separation on overall pressure distribution can be approached similarly using the Polhamus-like method.

The key factor in the development of the Polhamus-like formulation for the annular wings is to properly model the division of overall lift contribution from fully attached (potential) flow and vortex-induced flow. This is achieved by comparing a set of different theoretical models to the experimental data of the annular wing with

Received 22 February 2012; revision received 2 July 2012; accepted for publication 25 August 2012; published online 26 November 2012

*Assistant Professor, Research Centre for Modeling and Simulation.
†Assistant Professor, School of Mechanical and Aerospace Engineering. Senior Member AIAA.

an aspect ratio in the range of 1/3 to 3.0. It is found that some modified theoretical models closely agree with the experimental data whereas some do not. Subsequently, the concept of leading-edge suction analogy in [9] is extended to the aerodynamic estimation of the annular wings. The Polhamus theoretical model is applied to the experimental data in [7]. The variations of pressure and vortex-induced constants with aspect ratios are then generalized. The advantage of this approach is that the pressure-induced and vortex-induced performance can be approximated separately using an analytical relationship. Moreover, it provides an alternative approximation procedure as good as or better than the previous theoretical methods. Specifically, it is observed that the vortex lift constant is one-third of the value calculated in [11]. Moreover, the pressure constant is almost twice of the value for Zimmerman, inverse Zimmerman, rectangular, and elliptical planform shapes. Subsequently, the wind-tunnel testing of the annular wing designed at Nanyang Technological University for a ducted-fan UAV to operate in the low Reynolds number regime is carried out. The annular wing has a different airfoil cross section and aspect ratio than the one reported in the literature. The experimental aerodynamic coefficients are then fitted with the suction-based model developed earlier. An excellent agreement between the experimental and formulation developed is demonstrated.

II. Aerodynamic Estimation

The experimental data in [7], the details of which are repeated here, are used to study the aerodynamic behavior of the annular wings of various aspect ratios. Five aspect ratios are studied in [7]: 1/3, 2/3, 1.0, 1.5, and 3.0 with the Reynolds number variation $Re =$ 0.704–2.11 million. It should be noted that the aspect ratio is defined as the ratio of the diameter of the duct and its chord length. The projected planform area of all models is kept constant. The models are mounted at their quarter-chord points in a 6×6 ft wind-tunnel cross section. All of the annular wings tested have a Clark-Y airfoil cross section with a thickness-to-chord ratio of 11.7%. The data are presented in Fig. 1 for various aspect ratios. The investigation in this paper is focused in the prestall regime only, and, therefore, the data shown are only until the vicinity of the stall. As obvious from the general understanding, the lift-curve slope increases with the increase in aspect ratio. Moreover, the shift in the stall pattern to lower angles of attack at lower aspect ratios is also evident in Fig. 1. It should also be noted that the inherent nonlinear effects associated with small aspect ratios are negligible for annular wing planforms. The experimental data in [7] also show benign stall behavior for the annular wings.

In this section, existing methods used to predict the lift behavior of planar wings as a function of aspect ratio are applied to the annular wings following the modification in [6]. Following that, the Polhamus-like formulation based on suction analogy is developed to predict the lift pattern. The pressure-induced and vortex-induced lift behaviors are decoupled and studied separately. Subsequently, drag and moment behavior are also examined.

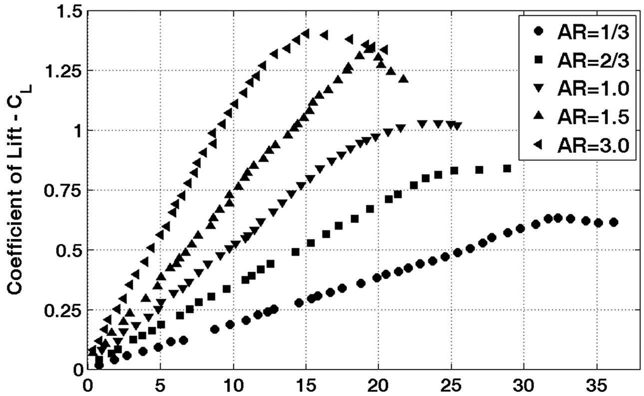

Fig. 1 Effect of aspect ratio on the variation of lift coefficient for annular wings [6].

A. Existing Techniques

The aerodynamic performance of the annular wings is generally quantified using their lift-curve slope ($C_{L\alpha}$). The lift-curve slopes of the experimental data in Fig. 1 are compared with a number of theoretical predictions in Fig. 2. The first comparison is based on the theoretical model proposed in [6] for the high-aspect ratio annular wings:

$$C_{L\alpha} = \frac{2\pi AR C_{l\alpha}}{4AR + C_{l\alpha}} \tag{1}$$

where $C_{l\alpha}$ is the lift-curve slope of the airfoil cross section. Fletcher [7] observed that the model is accurate for the aspect ratio greater than 2.4. It is evident from Fig. 2 that the model does not predict the experimental lift-curve slopes accurately except for the aspect ratio of 3.0.

The theoretical model suggested in [6] for the low-aspect ratio annular wings only matches the data for the aspect ratios of 1/3 and 2/3. There is no good relationship found between the experimental and the theoretical predictions for aspect ratios greater than 1. Ribner's [6] mathematical model for the low-aspect ratio case is given by

$$C_{L\alpha} = \pi AR \tag{2}$$

Ribner's [6] high- and low-aspect ratio annular wing lift-curve slope predictions are valid in the specified range only and cannot predict the complete range of aspect ratios between 1/3 and 3.0. Ribner [6] predicted that the lift of the annular wing is twice the lift of the flat elliptical wing of the same aspect ratio. The theoretical lift slope based on the Weissinger method as a function of aspect ratio is developed in [13] for different planform shapes. The lift-curve slopes of elliptical and rectangular wings obtained using this method are ad hocly multiplied by two (following the result in [6]) and compared with the experimental data in Fig. 2. It is observed that both trends fairly predict the experimental results.

Another theoretical model in [14] is also examined. This model is proposed for the low-aspect ratio planar planforms of less than 2, as follows:

$$C_{L\alpha} = \frac{2\pi AR}{2 + \sqrt{(AR^2/\eta^2)(1 + \tan \Lambda_{c/2}) + 4}} \tag{3}$$

where $\eta = C_{l\alpha}/2\pi$ and $\Lambda_{c/2}$ is the sweep back angle at midchord. For the annular wings, the sweep angle is treated as zero. The predicted values from this model are again multiplied by two, following the result in [6] and compared in Fig. 2. Interestingly, this model holds valid for low-aspect ratio wings but is not accurate for high-aspect ratio ones.

Finally, the formulation in [15] for aspect ratios less than 2.5 is studied:

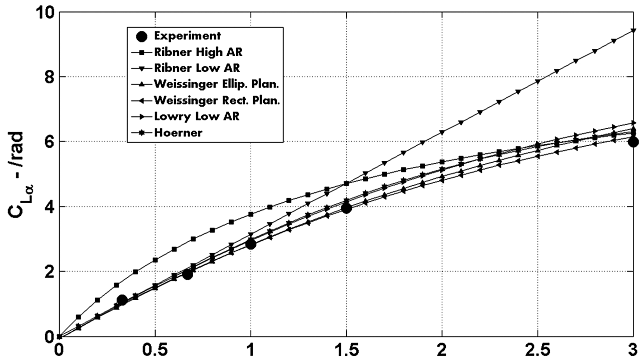

Fig. 2 Comparison of lift-curve slope with different theoretical models for various aspect ratios.

$$C_{L\alpha} = [36.5/AR + 2AR]^{-1} \tag{4}$$

Again, the values are multiplied by two and shown in Fig. 2. It can be observed that the model cannot accurately predict the lift-curve slope of the annular wings.

The methods described above to predict the lift-curve slope of the annular wings, as summarized in Fig. 2, are based on the linear lift-curve assumption, and, thus, they cannot capture the nonlinearities associated with the lift pattern. Therefore, in the next section, an alternative aerodynamic prediction method for the annular wing based on suction analogy that can capture a certain degree of nonlinearity in lift curve is proposed.

B. Leading-Edge Suction Analogy

This concept [9] is based on the fact that the total lift can be calculated as the sum of potential-flow lift, and the lift associated with the existence of leading-/side-edge vortices. The significant advantage of this approach is that the overall lift can be parameterized into pressure-induced and vortex-induced contributions, thereby, giving a better insight into the factors contributing to aerodynamic performance. It should be noted that the Polhamus method does not cater for the specific flow details, but rather it quantifies the suction force on the wing arising from the pressure required to keep the centrifugal force in equilibrium as the flow passes around the separation vortex.

The potential lift, $C_{L,p}$, can be calculated using the linear lifting line theory with the assumption of zero leading-edge suction as the streamlines will pass smoothly around the leading-edge separation bubble. Moreover, the flow will reattach itself on the upper surface of the wing after passing over the separation vortex. Therefore, the potential flow lift is decreased only by the loss of the leading-edge suction force. The potential lift can be expressed as [9]

$$C_{L,p} = K_p \sin\alpha \cos^2\alpha \tag{5}$$

where K_p is a constant to be determined using empirical techniques. Moreover, for small angles of attack, the expression is reduced to $C_{L,p} = K_p\alpha$, which shows that in this case K_p is analogous to the lift-curve slope obtained from the small angle theory.

Polhamus [9] presented explicit existence of the force generated by the vortex pattern emerging on the highly swept wings. These vortices help to reattach the flow on the upper surface of the wings, thereby, generating nonlinear lift pattern and significantly delaying stall angle of attack. Lamar [10] extended this concept to the side edges, such that the side-edge vortices that swirl to the upper surface of the wings also contribute to the vortex lift. The vortex lift equivalent to the leading-edge suction force acts normal to the wing surface, and the resolved component in the lift direction can be written as

$$C_{L,v} = K_v \cos\alpha \sin^2\alpha \tag{6}$$

where K_v is a constant. For planar wings, K_v generally does not vary much with aspect ratio, and its value can be approximated by π from studies conducted in [9,11,12] using various planform shapes. Torres and Mueller [11] obtain an excellent agreement between this model and experimental results using four different planform shapes (rectangular, Zimmerman, inverse Zimmerman, and elliptical) if the value of K_v is kept constant at π.

The total lift coefficient can be obtained by summing the potential lift [Eq. (5)] with the vortex lift [Eq. (6)] as follows: $C_L = K_p \sin\alpha\cos^2\alpha + K_v \cos\alpha\sin^2\alpha$A representative case of Eq. (7) is plotted in Fig. 3. It should be noted that the potential flow-based lift and vortex-induced lift add together to the actual total lift. In this representative case, the value of K_p is 1 and K_v is $\pi/3$.

Equation (7) is used to estimate the lift data of the annular wing for various aspect ratios in Fig. 4 by empirically finding the values of K_p and K_v The values of K_p as per estimation in [6] are set at twice the values of rectangular and elliptical wings found in [11]. For the value of K_v, a close agreement between the theoretical model and experimental data can only be demonstrated by setting its value to $\pi/3$ for all aspect ratios. This suggests that the vortex lift contribution for the annular wings is one-third of the other planar wing designs investigated in [11]. This is an interesting finding that has not been reported in literature to date.

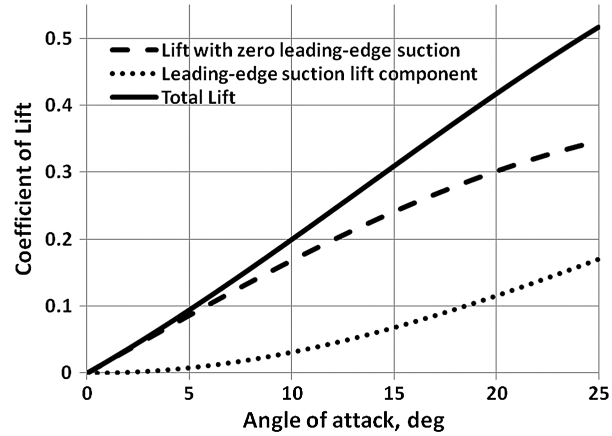

Fig. 3 Contribution of potential and vortex lift effects.

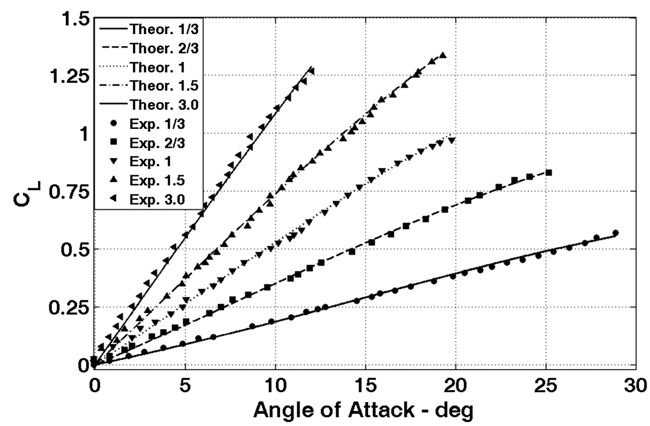

Fig. 4 Lift comparison between experimental and Polhamus suction analogy.

The value of K_p as a function of aspect ratio is plotted in Fig. 5. The variation of K_p from the aspect ratio 1/3 to 1.5 is linear in nature and can be expressed as

$$K_p = 2.821AR \tag{8}$$

For the complete aspect ratio range from 1/3 to 3.0, another relationship is found empirically:

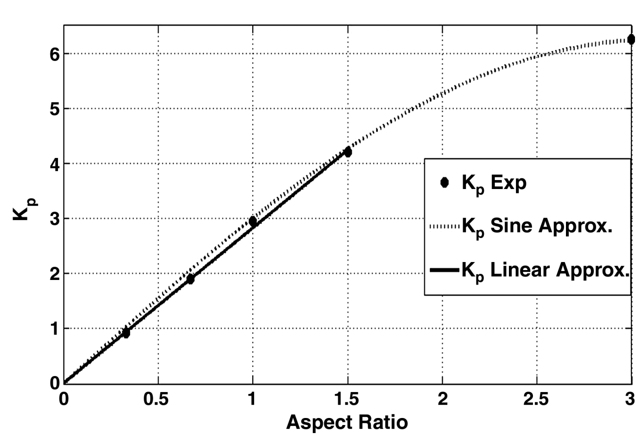

Fig. 5 Variation of K_p as a function of aspect ratio.

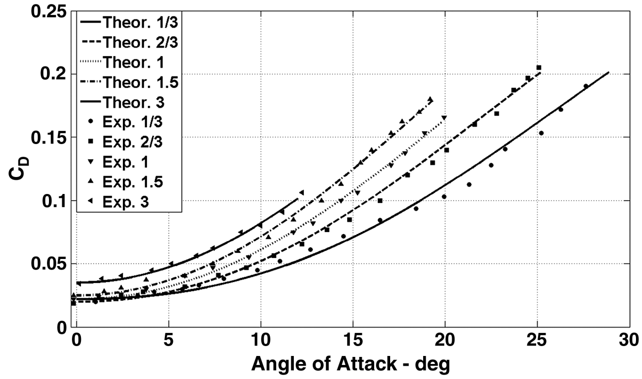

Fig. 6 Drag comparison with drag polar equation.

$$K_p = K_{p\,\max}\,\sin\left(\frac{AR}{2}\right) \tag{9}$$

where $K_{p\,\max} = 6.25$. Note that in any of these approximations, the values of K_p are almost twice that of the values of different planar configurations in [11].

Lamar [10] extended the suction analogy to calculate drag and pitching moments of the wings. The drag approximation used is

$$C_D = C_{Do} + K_p \sin^2\alpha\,\cos\alpha + K_v \sin^3\alpha \tag{10}$$

However, it is observed that the drag relationship does not predict the drag behavior accurately. This is consistent in [11]. The drag is fairly well predicted by a more renowned drag polar equation. The comparison is shown in Fig. 6:

$$C_D = C_{Do} + KC_L^2 \tag{11}$$

The induced drag coefficient, K, is plotted as a function of aspect ratio in Fig. 7. It is observed that the trend closely follows the following power law:

$$K = a(AR)^b \tag{12}$$

where $a = 0.15$ and $b = -1.237$ The value of K for the annular wings is compared with other planform shapes in Fig. 7. It can be observed that the value of K is significantly lower for the annular wings as compared to various planar shapes.

Finally, the coefficient of pitching moment can be approximated using the formula proposed in [10]:

$$C_M = x_p K_p\,\sin\alpha\,\cos\alpha + x_e K_v \sin^2\alpha \tag{13}$$

The experimental data are compared with the theoretical model in Fig. 8, which demonstrates the good agreement.

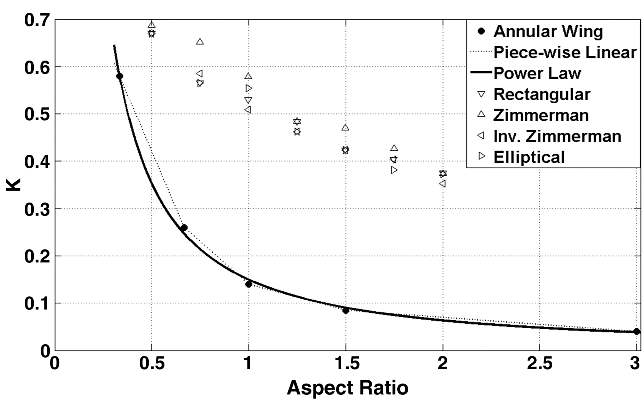

Fig. 7 Parameter K as a function of aspect ratio.

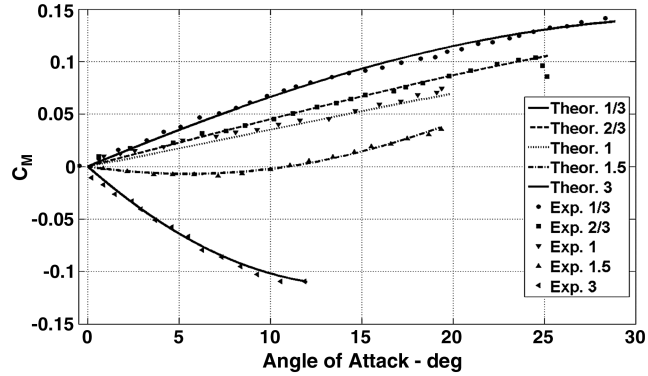

Fig. 8 Pitching moment comparison between experimental and Polhamus/Lamar suction analogy.

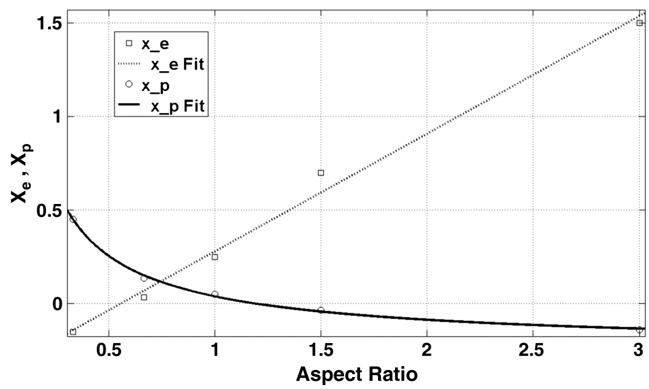

Fig. 9 Parameters x_e and x_p as function of aspect ratio.

The parameter x_p and x_e are referenced at the quarter-chord point. The approximation for x_p is accurately approximated with

$$x_p = aAR^b + c \tag{14}$$

where $a = 0.3$, $b = -0.786$, $c = -0.26$. The trend is plotted in Fig. 9. On the other hand, the variation of the parameter x_e with aspect ratio is found to be linear, as follows:

$$x_e = p_1 AR + p_o \tag{15}$$

where $p_1 = 0.63$, $p_o = -0.35$.

III. Validation Studies

A. Description of the Model

The platform used to further validate the results of this study is a conventional annular wing as shown in Fig. 10. The annular wing is circular with the same cross section. The aspect ratio of the wing is 1.62. The dimensional details of the model include a span of 0.235 m and a chord length of 0.145 m. The planform area of the annular wing is 0.034 m^2, which is different from the setup in [7]. The airfoil of the

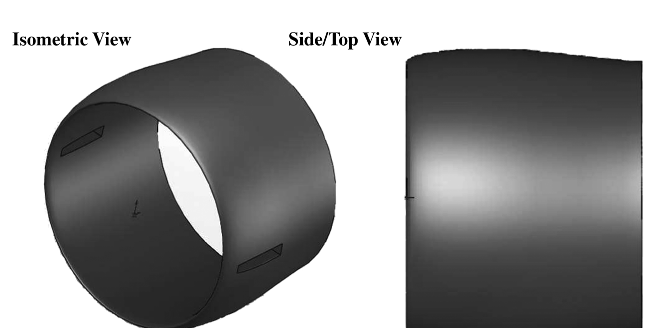

Fig. 10 Isometric and side/top view of annular wing.

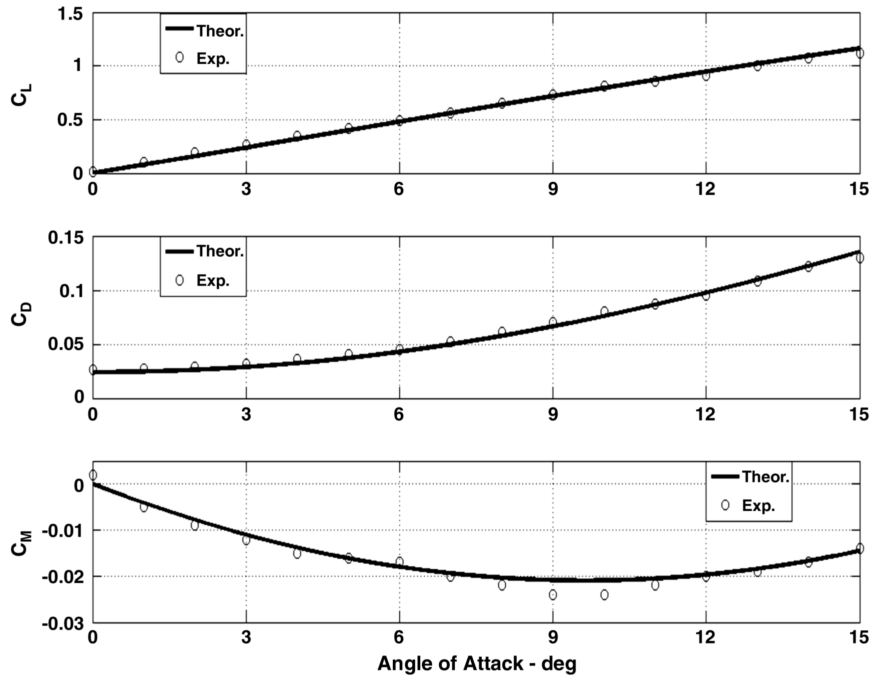

Fig. 11 Comparison between experimental and theoretical lift data.

wing is Eppler 180 for the low Reynolds number applications with the thickness-to-chord ratio of 8.59%. The wind-tunnel model is primarily made of an acrylonitrile butadiene styrene honeycomb structure commonly known to us as Lego® blocks material. The model is constructed through laser machining.

B. Aerodynamic Evaluation

The aerodynamic characteristics of the annular wing are obtained through wind-tunnel testing. The experiment is carried out in a low-turbulence closed-circuit wind tunnel at Nanyang Technological University. It has a contraction ratio of 9 and a rectangular inlet contraction cone designed specifically for low-turbulence intensity, as well as three antiturbulence screens with different meshes. The test section is 2 m long with a rectangular cross-sectional area of $0.78 \text{ m} \times 0.72 \text{ m}$. The model is mounted at the quarter-chord location on a six-component internal balance. The Data Acquisition, Reduction, and Control System is based on a National Instruments platform and Lab-View®-based software. Force and moment coefficient presented in this work have been corrected for wind-tunnel blockage effects (solid blockage, wake blockage, and streamline curvature) according to the techniques presented in [16]. It was found that the blockage effects are minimum in the setup. Similarly, it was found that no hysteresis is present in the setup. The test is conducted at the Reynolds number of approximately $\text{Re} = 0.2 \times 10^6$ corresponding to the velocity of 15 m/s.

C. Comparison with Theoretical Model

The aerodynamic coefficients obtained from the experiment are presented in Fig. 11 and compared with the formulations developed. For lift, the approximation of K_p is taken from Eq. (9) for the aspect ratio of 1.62. The value of K_v is kept constant at $\pi/3$. An excellent agreement is demonstrated for the lift in the prestall regime. The validity of this model holds up to an angle of attack of 15 deg, which is the maximum angle of attack done in the experiment.

The coefficient of drag is fitted with the classical drag polar equation. The induced drag constant, K, is fitted with the value of 0.06. The coefficient of moment experimental data is fitted with the theoretical model discussed in Eqs. (13–15). The trend adequately follows experimental data at a low and high angle of attack. However, the fitting around the 9 deg angle of attack does not accurately predict experimental values. The cause is probably attributed to the poor approximation of x_e in Eq. (15).

The comparison between the formulation developed and the data from the experiment is found to be fairly accurate. The minor differences can be attributed to multiple factors like geometry, Reynolds number, and experimental uncertainty, etc. Therefore, the proposed formulation can be used for a quick estimation of the aerodynamic forces and the moment for the annular wings between the aspect ratio of 0.5 and 3.0.

IV. Conclusions

An alternative method based on leading-edge suction analogy is proposed for the aerodynamic estimation of annular wings. The significant advantage of this approach is that the overall lift can be separated into pressure-induced and vortex-induced contributions, thereby, giving a better insight into the aerodynamic performance. The experimental data for five annular wings with varying aspect ratios are used in the analytical formulation development. The formulation developed suggests that the vortex lift for the annular wings is about one-third of that from planar configurations with the same aspect ratio, whereas the pressure-induced lift is consistently about twice as high as that of planar shapes (rectangular, elliptical, Zimmerman, and inverse Zimmerman) for the same aspect ratio, which is conformed to the previous result in the literature. For the coefficient of drag, the classical drag polar equation is found to be more appropriate for the annular wing drag prediction across a varying aspect ratio and angle of attack. The coefficient of the pitching moment formulation is shown to yield an acceptable prediction of the moment coefficient as well. The aerodynamic predictions using the formulations developed are further shown to be accurate by experimental validation using an annular wing of different geometric details.

References

[1] Fleming, J., Jones, T., Ng, W., Gelhausen, P., and Enns, D., "Improving Control System Effectiveness for Ducted Fan VTOL UAVs Operating in Crosswinds," *2nd AIAA "Unmanned Unlimited" Conference and Workshop & Exhibit*, AIAA, San Diego, CA, 2003; also AIAA Paper 2003-6514.

[2] Ko, A., Gelhausen, P., and Ohanian, O., "Ducted Fan UAV Modeling and Simulation in Preliminary Design," *AIAA Modeling and Simulation Technologies Conference and Exhibit*, AIAA, Hilton Head, SC, 2003; also AIAA Paper 2007-6375.

[3] "Fan-Tail System," ST Aerospace, http://www.staero.aero/www/keyoffering.asp?serkeyid=ODAwMDAwMDU [retrieved 23 Oct. 2011].
[4] Schaefer, C., and Baskett, L., "GoldenEye: The Clandestine UAV," *2nd AIAA "Unmanned Unlimited" Conference and Workshop and Exhibit*, AIAA, San Diego, CA, 16–18 Sept. 2003; also AIAA Paper 2003-6634.
[5] Traub, L. W., "Experimental Investigation of Annular Wing Aerodynamics," *Journal of Aircraft*, Vol. 46, No. 3, 2009, pp. 988–996. doi:10.2514/1.39822
[6] Ribner, H. S., "The Ring Airfoil in Nonaxial Flow," *Journal of Aeronautical Sciences*, Vol. 14, No. 9, 1947, pp. 529–530.
[7] Fletcher, H. S., "Experimental Investigation of Lift, Drag and Pitching Moment of Five Annular Airfoils," *National Advisory Committee for Aeronautics*, NACA TN 4117, 1957.
[8] Demasi, L., "Investigation on Conditions of Minimum Induced Drag of Closed Wing Systems and C-Wings," *Journal of Aircraft*, Vol. 44, No. 1, 2007, pp. 81–99. doi:10.2514/1.21884
[9] Polhamus, E. C., "A Concept of the Vortex Lift of Sharp-Edge Delta Wings Based on a Leading-Edge-Suction Analogy," NASA TN D-3767, 1966.
[10] Lamar, J. E., *Extension of Leading-Edge-Suction Analogy to Wing with Separated Flow Around the Side Edges at Subsonic Speeds*, NASA TR R-428, 1974.
[11] Torres, G. E., and Mueller, T. J., "Low-Aspect-Ratio Wing Aerodynamics at Low Reynolds Numbers," *AIAA Journal*, Vol. 42, No. 5, 2004, pp. 865–873. doi:10.2514/1.439
[12] Okamoto, M., and Azuma, A., "Aerodynamic Characteristics at Low Reynolds Numbers for Wings of Various Planforms," *AIAA Journal*, Vol. 49, No. 6, 2011, pp. 1135–1149. doi:10.2514/1.J050071
[13] DeYoung, J., "Theoretical Additional Span Loading Characteristics of Wings with Arbitrary Sweep, Aspect Ratio and Taper Ratio," NASA TN 1491, 1947.
[14] Lowry, J. G., and Polhamus, E. C., "A Method for Predicting Lift Increments Due to Flap Deflection at Low Angles of Attack in Incompressible Flow," NACA TN 3911, 1957.
[15] Hoerner, S. F., and Borst, H. V., *Fluid-Dynamic Lift*, Hoerner Fluid Dynamics, Brick Town, NJ, 1975, pp. 9-8, 9-9, 17-5, and 20-8.
[16] Barlow, J. B., *Low-Speed Wind Tunnel Testing*, 3rd ed., Wiley, New York, 1999, pp. 412–420.

A. Naguib
Associate Editor